

Lesson 3-5: Lines in the Coordinate Plane

Back to algebra

So we're back to algebra. I thought this was geometry. Well we are getting back to algebra and this is geometry. Those two subjects are not as far apart as you may imagine.

Do you remember the guy in his hospital bed that was watching the fly buzz around the ceiling? His name was Descartes and he invented the Cartesian coordinate system. When he invented it, he also invented a way for us to evaluate geometric figures and relationships using algebra. His method came to be called analytic geometry and eventually gave birth to calculus. So when you are sitting in your calculus class a few years from now, you know who you have to thank!

Lines again huh?

How many points does it take to define a line? That's right; two (postulate 1-1). If you remember your algebra, if you have two points, you can write an equation that describes that line. That equation is called a linear equation.

Ways we can write linear equations

There are three basic forms we can write linear equations in:

1. Slope-intercept form ... $y = mx + b$
2. Standard form ... $Ax + By = C$
3. Point-slope form ... $y - y_1 = m(x - x_1)$

Let's look at each in more detail. First I want to make sure you remember what slope is.

Slope

Slope is basically how tilted (and in which direction) a line is. A very common way of describing it is "rise over run." Rise is the vertical change (in the y direction). Run is the horizontal change (in the x direction). So, slope m for a line going through two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Slope} = \frac{\text{vertical.change}}{\text{horizontal.change}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Slope-intercept form

The slope-intercept form is $y = mx + b$ where m is the slope and b is the y -intercept. The y -intercept is where the line crosses the y -axis. Given the slope-intercept form, the y -intercept is at $(0, b)$.

According to Postulate 1-1 we need two points to determine a line. The y -intercept gives us one at $(0, b)$. If we use the slope we can determine another point. Start at the y -intercept. Look at the slope in fraction form:

- Do "rise": if the slope is positive, move up the number of units in the numerator. If it is negative, move down the number of units in the numerator.

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- Next do “run”: move to the right the number of units in the denominator.
- The ending point will be the 2nd point.

Example – Pg 155, #4

Graph the line $y = -\frac{5}{3}x + 2$:

$$m = -\frac{5}{3},$$

$$b = 2 \text{ or } (0, 2)$$

Standard form

The standard form of a linear equation is $Ax + By = C$ where A , B , and C are real numbers, and A and B are not both zero.

The easiest way to find two points in standard form is to determine the x and y intercepts:

- To find the x -intercept, substitute 0 for y and solve for x .
- To find the y -intercept, substitute 0 for x and solve for y .

Example – Pg 155, #10

Graph the line $1.2x + 2.4y = 2.4$ using intercepts:

$$y \text{ intercept: } 1.2(0) + 2.4y = 2.4 \text{ or } y = 1 \dots (0, 1)$$

$$x \text{ intercept: } 1.2x + 2.4(0) = 2.4 \text{ or } x = 2 \dots (2, 0)$$

Sometimes you will need to convert from standard form to slope-intercept form. To do so, simply solve for y .

Example – Pg 155, #16

Write the equation in slope-intercept form and graph it:

$$\frac{3}{4}x - \frac{1}{2}y = \frac{1}{8}$$

$$6x - 4y = 1$$

$$-4y = -6x + 1$$

$$y = \frac{3}{2}x - \frac{1}{4}$$

$$m = \frac{3}{2}, b = -\frac{1}{4} \text{ or } (0, -\frac{1}{4})$$

Point-slope form

The point-slope form is used when you know the slope and one point (not the y -intercept). It is $y - y_1 = m(x - x_1)$.

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Example – Pg 155, #20

Given the point $A(-2,-6)$, and slope -4 , write an equation in point-slope form.

$$y - (-6) = (-4)(x - (-2))$$
$$y + 6 = -4(x + 2)$$

Example – Pg 155, #28

Write an equation in point-slope form of the line:

$$P(8, 10), Q(-4, 2)$$
$$y - 10 = \left(\frac{10 - 2}{8 - (-4)}\right)(x - 8)$$
$$y - 10 = \frac{8}{12}(x - 8)$$
$$y - 10 = \frac{2}{3}(x - 8)$$

Horizontal line equation

What is the slope of a horizontal line? The line is “flat” so there is no rise (change in y) hence the slope m is 0. Therefore the equation of a horizontal line through (a, b) is:

$$y = 0x + b \text{ or } y = b$$

Vertical line equation

What is the slope of a vertical line? The line is straight up and down, so there is no run (change in x); the denominator of the slope would be 0... which is a big fat no-no. Can't divide by zero. So we say the slope of a vertical line is undefined. The equation of a vertical line through (a, b) is:

$$x = a$$

Example – Pg 155, #32

Write equations for (a) the horizontal line and (b) the vertical line containing the given point:

$$E(6, 4)$$

a) $y = 4$
b) $x = 6$

Assign homework

p. 155 #1-43 odd, 48-51, 53, 55, 61-64